TIMING ESTIMATION FOR QUASI-SYNCHRONOUS SDMA SYSTEMS

Shao-Dan Ma, Tung-Sang Ng, Fellow, IEEE

Department of Electrical and Electronic Engineering
The University of Hong Kong, Pokfulam Road, Hong Kong, China
Email: {sdma,tsng}@eee.hku.hk

ABSTRACT

In this paper, timing estimation for quasi-synchronous SDMA systems is considered. The problem arises in the uplink of a mobile communication system where precise synchronization among all mobiles is very difficult to be achieved and signals of all mobiles are time-aligned at the receiver within a small synchronization window. An efficient timing estimation method is proposed. It can simultaneously estimate the channel responses and the time delays which are required for signal detection. Simulation results show that the time delays can be accurately estimated and the performance of MMSE multi-user detection with the proposed method is very close to that with the perfect timing and channel information.

I. INTRODUCTION

Space Division Multiple Access (SDMA), with the potential to greatly increase the system capacity without extra bandwidth, has been considered as a major technique in multi-user wireless communications. It can be directly applied to the uplink of a cellular mobile communication system, since mobiles are generally distributed at different locations and the base station can separate the signals with the use of multiple receive-antennas and signal processing techniques. Many signal detection algorithms for SDMA systems have been proposed [1]–[7]. However, most of them have a restriction that perfect synchronization among all users is required, which is very difficult in practical situations, especially in cellular mobile communication systems due to different locations of mobiles and signals propagating through different fading channels. In this paper, we consider quasi-synchronous SDMA systems, in which signals of all users are time-aligned at the receiver within a small synchronization window. It does not need perfect synchronization and thus is more realistic in practice. To the best of our knowledge, very few work has been done for the signal detection in quasi-synchronous SDMA systems.

In quasi-synchronous systems, the knowledge of the time delays is required for signal detection. The accuracy of the time delay estimation has a profound effect on the performance of signal detection. In [8], two pilot-based timing estimation algorithms based on sliding correlation and vector orthogonalization have been proposed. For the sliding correlation method, the time delays cannot be precisely estimated when the difference between any two delays is less than two symbol periods. As for the vector orthogonalization method, the need to estimate the noise subspace of the received signal matrix increases its computation-complexity and therefore imposes a limit on its application to practical systems. Another limitation of these two methods is the requirement of long training sequences, which reduces the bandwidth efficiency. Another pilot-based timing estimation algorithm was proposed in [9]. This algorithm respectively estimated each user's time delay based on a least-squares-constant-modulus-criterion cost function. It is thus very complex. In this paper, a timing estimation method without the previous restrictions is proposed for quasi-synchronous SDMA systems. It can simultaneously estimate the time delays and the channel responses with short training sequences. Simulation results demonstrate that the proposed method is efficient and the performance of MMSE multi-user detection with the proposed method is very close to that with the perfect timing and channel information.

The paper is organized as follows. In Section II, the quasi-synchronous SDMA system model is formulated. In Section III, the timing estimation algorithm is proposed. Section IV presents the simulation results and conclusion is given in Section V.

II. SYSTEM MODEL

The uplink of a cellular mobile communication system with SDMA technique can be considered as a quasi-synchronous SDMA system with P users each of which transmits a signal via a single antenna, and with a receiver having M receive-antennas. We employ discrete-time representation for the SDMA system and different users may have different time delays as shown in Fig. 1. Let $d_i \geq 0$, $i \in \{1, 2, \cdots, P\}$, be the time delay of the user signal and takes on an integer value. Denote d_{PU} as the synchronization window of the system, namely,

0-7803-8549-7/04/$20.00 © 2004 IEEE

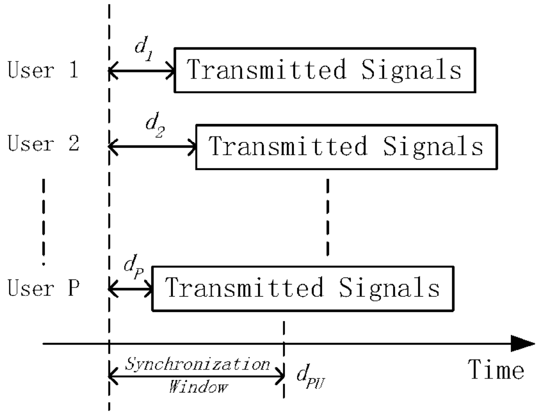

Fig. 1. Quasi-synchronous transmission scheme

the maximum time delay in the system. $d_i \leq d_{PU}$, $i \in \{1,\ 2,\ \cdots,\ P\}$. All the time delays are assumed to be not larger than the synchronization window. In the special case of synchronized transmission, the delays for all users are equal to zero. The channel response from the transmit-antenna of the ith user to the jth receive-antenna at the receiver is denoted as h_{ij}. The jth receive-antenna output at time n is

$$y_j(n) = \sum_{i=1}^{P} h_{ij} x_i(n-d_i) + w_j(n),\ j = 1, 2, \cdots, M, \quad (1)$$

where $w_j(n)$ is the noise at the jth receive-antenna and $x_i(n)$ is the input signal of the ith user at time n. The received signal vector, $\mathbf{y}(n)$, is therefore

$$\mathbf{y}(n) = \mathbf{h}\mathbf{x}(n) + \mathbf{w}(n), \quad (2)$$

in which

$$\begin{aligned} \mathbf{y}(n) &= [y_1(n),\ y_2(n),\ \ldots,\ y_M(n)]^T \\ \mathbf{x}(n) &= [x_1(n-d_1), x_2(n-d_2), \ldots, x_P(n-d_P)]^T \\ \mathbf{w}(n) &= [w_1(n),\ w_2(n),\ \ldots,\ w_M(n)]^T \\ \mathbf{h} &= \begin{pmatrix} h_{11} & h_{21} & \cdots & h_{P1} \\ h_{12} & h_{22} & \cdots & h_{P2} \\ \vdots & \vdots & \vdots & \vdots \\ h_{1M} & h_{2M} & \cdots & h_{PM} \end{pmatrix} \end{aligned} \quad (3)$$

In (3), $(\cdot)^T$represents matrix transpose.

III. TIMING ESTIMATION

In order to perform the signal detection for all users in quasi-synchronous systems, the knowledge of the time delays d_i, $\{1,\ 2,\ \cdots,\ P\}$, is a necessity. Since blind methods cannot estimate the time delays, some training symbols embedded in the signals are required. In this section, a pilot-based method is proposed to estimate the delays.

To simplify the derivation of the algorithm, zero noise is first assumed. The received signal vector $\mathbf{y}(n)$ can be expressed as

$$\mathbf{y}(n) = \mathbf{h}\mathbf{x}(n). \quad (4)$$

From (4), we notice that the delay information is embedded in the transmitted signal vector . Traditionally, time delays are estimated based on system model (4), e.g. the two timing estimation methods proposed in [8]. However, they have many restrictions which have been presented in Section I. In the following, we construct an equivalent system model and transform the delay information into the remodelled channel matrix. Then the time delays can be estimated from the remodelled channel matrix.

Denote the signal vector with $d_{PU}+1$ sampled input signals of all users as

$$\overline{\mathbf{x}}(n) = [x_1(n), \ldots, x_P(n), \ldots, x_1(n-d_{PU}), \ldots, x_P(n-d_{PU})]^T. \quad (5)$$

An equivalent system model can be designed as

$$\mathbf{y}(n) = \mathbf{h}\mathbf{x}(n) = \mathbf{H}\overline{\mathbf{x}}(n), \quad (6)$$

In (6), $\mathbf{H}$ is an $M \times (d_{PU}+1)P$ matrix, where the $(d_i P + i)$th column is the ith, $i \in \{1,\ 2,\ \ldots,\ P\}$, column of the channel matrix $\mathbf{h}$ and other columns are all-zero columns. From the equivalent system model on the special channel matrix $\mathbf{H}$ defined in (6), we observe that the remodeled channel matrix $\mathbf{H}$ has only P non-zero columns, each of which is on the position $(d_i P + i)$, $i \in \{1,\ 2,\ \ldots,\ P\}$. The delay information is thus embedded in the remodeled channel matrix $\mathbf{H}$.

Assume that $(d_{PU}+1)P$ sampled received signal vectors are collected. Denote the observed signal vectors as an $M \times (d_{PU}+1)P$ matrix given by

$$\mathbf{Y}_{(d_{PU}+1)P}(n) = [\mathbf{y}(n), \mathbf{y}(n+1), \ldots, \mathbf{y}(n+(d_{PU}+1)P-1)]\,. \quad (7)$$

It follows that

$$\mathbf{Y}_{(d_{PU}+1)P}(n) = \mathbf{H}\mathbf{X}_{(d_{PU}+1)P}(n). \quad (8)$$

In (8), $\mathbf{X}_{(d_{PU}+1)P}(n)$ is a $(d_{PU}+1)P \times (d_{PU}+1)P$ matrix as

$$\mathbf{X}_{(d_{PU}+1)P}(n) = [\overline{\mathbf{x}}(n), \overline{\mathbf{x}}(n+1), \ldots, \overline{\mathbf{x}}(n+(d_{PU}+1)P-1)]\,. \quad (9)$$

Suppose that $(d_{PU}+1)P + d_{PU}$ pilot symbols are inserted into each user's signal and $\mathbf{X}_{(d_{PU}+1)P}(n)$ contains the pilot symbols. Then the least square estimation of the special matrix $\mathbf{H}$ can be derived as [10]

$$\hat{\mathbf{H}} = \mathbf{Y}_{(d_{PU}+1)P}(n)\mathbf{X}_{(d_{PU}+1)P}(n)^* (\mathbf{X}_{(d_{PU}+1)P}(n)\mathbf{X}_{(d_{PU}+1)P}(n)^*)^\dagger \quad (10)$$

where $(\cdot)^*$denotes conjugate transpose and $(\cdot)^\dagger$ is pseudo inverse.

With the estimated matrix $\hat{\mathbf{H}}$, a maximum selection algorithm to simultaneously estimate the delays d_i, $i \in \{1, 2, \ldots, P\}$, and the channel response matrix $\mathbf{h}$ can be derived as follows. Recall that $\mathbf{H}$ is the $M \times (d_{PU}+1)P$ matrix, the (d_iP+i)th column of which is the ith column of the channel response matrix $\mathbf{h}$ while other columns are all-zero columns. The norm of the (d_iP+i)th column in $\mathbf{H}$ must be the largest among the (m_iP+i)th columns, $m_i \in \{0, 1, \ldots, d_{PU}\}$, because other columns among these $(d_{PU}+1)$ columns are all-zero columns. With this special structure property, we can group the (m_iP+i)th columns, $m_i \in \{0, 1, \ldots, d_{PU}\}$. P groups are therefore constructed. The column with the largest norm can be selected from each group. Then the estimated time delay $\hat{d}_i$, $i \in \{1, 2, \ldots, P\}$, is equal to the parameter m_i of the selected column in the ith group. At the same time, the selected P columns make up of the estimate of the channel response matrix $\mathbf{h}$.

For the noise case, the special channel matrix $\mathbf{H}$ can be estimated by MMSE method. As the estimation is based on training sequences, we expect that the noise contribution has little impact.

After estimating the time delays and the channel response matrix $\mathbf{h}$, the multi-user detector based on the MMSE criterion can be easily designed. It yields

$$\hat{\mathbf{x}}(n) = \hat{\mathbf{h}}^*(\hat{\mathbf{h}}\hat{\mathbf{h}}^* + \frac{1}{SNR}\mathbf{I})^{-1}\mathbf{y}(n), \tag{11}$$

where $\mathbf{I}$ is an identity matrix and the signal-to-noise-ratio, (*SNR*), is

$$SNR = \frac{\sum_{j=1}^{M} E\left\{\left|\sum_{i=1}^{P} h_{ij}x_i(n-d_i)\right|^2\right\}}{\sum_{j=1}^{M} E\left\{|w_j(n)|^2\right\}}. \tag{12}$$

IV. SIMULATION RESULTS

In this section, a quasi-synchronous SDMA system with $P = 3$ users and $M = 3$ receive-antennas is used as an example to investigate the performance of the proposed method. QPSK modulation is applied in this system. The synchronization window d_{PU} is set to 8. Assume that the time delays of all users are as follows: $d_1 = 2$, $d_2 = 4$, and $d_3 = 1$ (arbitrarily selected as illustration). The randomly generated channel response matrix $\mathbf{h}$ is

$$\mathbf{h} = \begin{bmatrix} 0.8644+0.1684i & 0.8735-0.5523i & -1.1027-0.6149i \\ 0.0942-1.9654i & -0.4380-0.8197i & 0.3962-0.2546i \\ -0.8519-0.7443i & -0.4297+1.1091i & -0.9649-0.2698i \end{bmatrix}. \tag{13}$$

Note that the three columns in the channel response matrix $\mathbf{h}$ are corresponding to the $(2 \times 3 + 1)$th, $(4 \times 3 + 2)$th, $(1 \times 3 + 3)$th columns in the special channel matrix $\mathbf{H}$. Other columns in $\mathbf{H}$ are all-zero columns. In the example, $(d_{PU}+1)P + d_{PU}$ pilot symbols are exploited and the training sequence matrix $\mathbf{X}_{(d_{PU}+1)P}(n)$ is selected to have full rank.

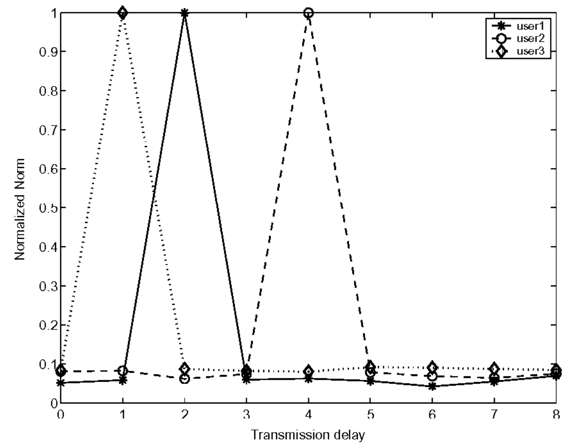

Fig. 2. Timing estimation for quasi-synchronous SDMA systems

In Fig. 2, the solid line shows the normalized norms of the (m_iP+1)th, $m_i \in \{0, 1, \ldots, d_{PU}\}$, columns in the estimated matrix $\hat{\mathbf{H}}$. The dashed line indicates the normalized norms of the (m_iP+2)th, $m_i \in \{0, 1, \ldots, d_{PU}\}$, columns and the dot line is corresponding to the normalized norms of the (m_iP+3)th, $m_i \in \{0, 1, \ldots, d_{PU}\}$, columns. Each group norms are normalized by the maximum norm in the group. The simulation is performed under the condition that $SNR = 20dB$. It is apparent that the peaks are very sharp and the time delays can be accurately estimated by the positions of the peaks even if the condition that the difference between any two delays is not less than two symbol periods is not satisfied. Note that this condition is required for the sliding correlation method in [8].

Fig. 3 shows the performance of the MMSE multi-user detection with the proposed timing estimation method. For comparison, the MMSE multi-user detection with perfect channel and timing information (MMSE-ideal) is also implemented. The BER (bit error rate) is averaged over 1000 Monte Carlo runs, each of which consists of 500 symbols. It is shown that the performance of MMSE multi-user detection with this timing estimation is very close to that with the perfect timing and channel information. Note that the performance of the MMSE-ideal algorithm is the best that can be achieved for linear multi-user detection algorithm as the channel response matrix and time delays are assumed known exactly. It further demonstrates the accuracy of the proposed timing estimation method.

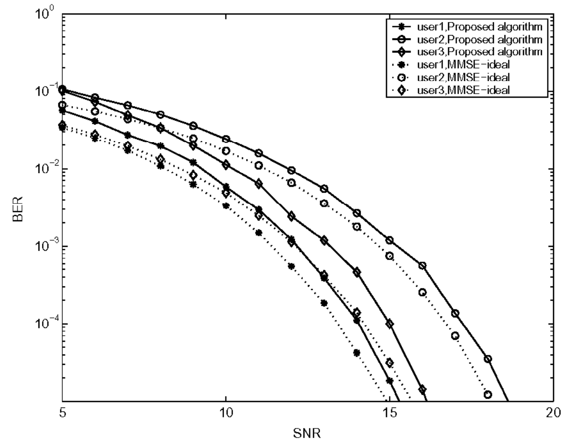

Fig. 3. Multi-user detection with the proposed timing estimation for quasi-synchronous SDMA systems

V. CONCLUSION

An efficient timing estimation method for quasi-synchronous SDMA systems is proposed in this paper. The maximum selection algorithm among the norms of each grouped column block in the estimated special matrix is introduced to simultaneously estimate the time delays and the channel response matrix. Only the synchronization window d_{PU} is required in the proposed method. Simulation results show that the proposed method can accurately estimate the time delays and the performance of MMSE multi-user detection with this timing estimation is very close to that with the perfect timing and channel information.

ACKNOWLEDGMENT

This work was supported by the Hong Kong Research Grants Council (Grant No. 7168/03E).

VI. REFERENCES

[1] Q. Yin and M. Yao, "Spatial signature estimation in cyclic cumulant domain and multiuser signal separation in uplink SDMA," *IEEE Proc. ICASSP'99,* vol. 5, pp. 2861 -2864, 1999

[2] I. Kostanic and W. Mikhael, "Blind source separation technique for reduction of co-channel interference," *Electronics Letters,* vol. 38, pp. 1210-1211, Sep. 2002

[3] A. Yeredor, "Non-orthogonal joint diagonalization in the Least-Squares sense with application in blind source separation," *IEEE Trans. Signal Processing*, vol. 50, pp. 1545-1553, July 2002

[4] E. Moreau, "A generalization of joint-diagonalization criteria for source separation," *IEEE Trans. Signal Processing,* vol. 49, pp. 530-541, Mar. 2001

[5] A. Belouchrani, K. Abed-Merain, J. F. Cardoso and E. Moulines, "A blind source separation technique using second-order statistics," *IEEE Trans. Signal Processing,* vol. 45, pp. 434-444, Feb. 1997

[6] Y. Li and K. J. R. Liu, "Adaptive blind source separation and equalization for multiple-input/multiple-output systems," *IEEE Trans. Inform. Theory,* vol. 44, pp. 2864-2876, Nov. 1998

[7] Q. Li, E. W. Bai and Z. Ding, "Blind source separation of signals with known alphabets using ε-Approximation algorithms," *IEEE Trans. Signal Processing,* vol. 51, pp. 1-10, Jan. 2003

[8] T. Nishimura, Y. Tanabe, T. Ohgane, Y. Ogawa, Y. Doi and J. Kitakado, "Spatial domain interference canceler using multistage adaptive array with precise timing estimation," *IEEE Proc. VTC'2000-Fall,* vol. 2, pp. 719-724, Sept. 2000

[9] A. M. Kuzminskiy, D. Hatzinakos, "Semi-blind symbol synchronization for antenna array signal processing in short burst multiuser SDMA systems," *IEEE Workshop on SPAWC '99*, pp. 142 - 145, May 1999

[10] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory,* New Jersey: Prentice-Hall, 1993.